Phase Transitions

Examples and applications

Boiling of water, Melting of ice, Sublimation of napthalene balls, Ferromagnet – Paramagnet transition in some metals (e.g. Iron), Melting & freezing of metals to form various objects (e.g. uteneils, etc).

ORDER PARAMETER Some macroscopic quantity that is zero in one phase and non-zero in the other phase, across a phase transition.

Examples of order parameters: $\rho - \rho_{gas}$ (liquid – gas transition)

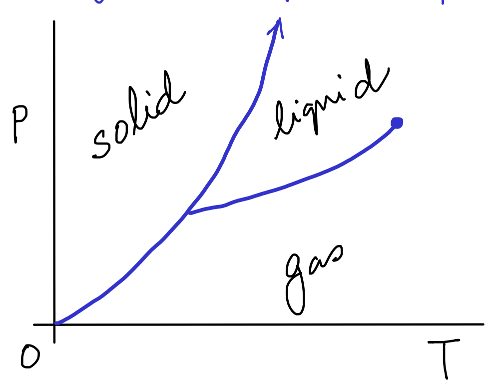
Magnetization (ferromagnet – paramagnet transition)

First order phase transition: This kind of phase transition is associated with a discontinuous change in the order parameter.

- The phases on the two sides of the phase transition are radically different.
- In some cases, this is associated with a non-zero value of the latent heat.

Examples

① Melting / Freezing, Evaporation / Condensation, Sublimation / Deposition.

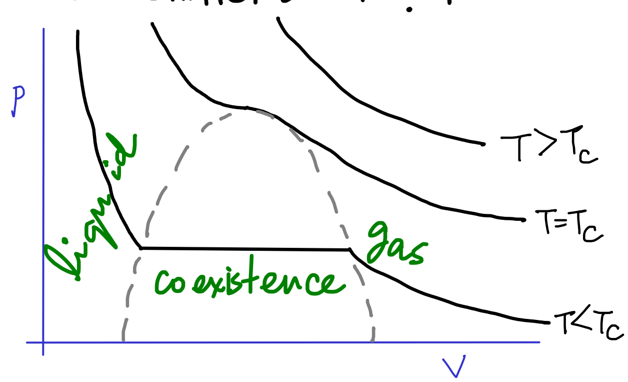


Phase diagram of a simple material (e.g, argon)

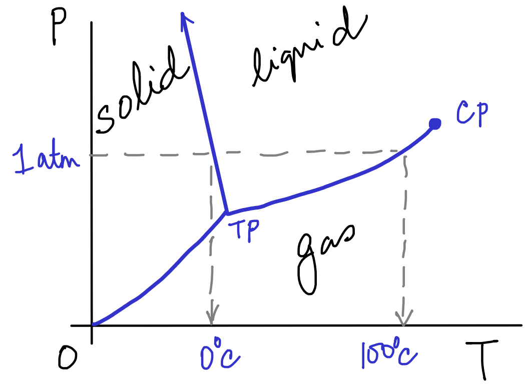
- First order phase transitions occur when the boundaries between different phases are crossed.
- The liquid-gas boundary ends abruptly at a CRITICAL POINT

P-V phase diagram

The solid phase which can be seen at lower volume (higher density) is not drawn here.



Note: For water, the solid liquid boundary has opposite slope in the p-T phase diagram.



Triple Point (TP)

0.01°C, 0.006 atm

Critical Point (CP)

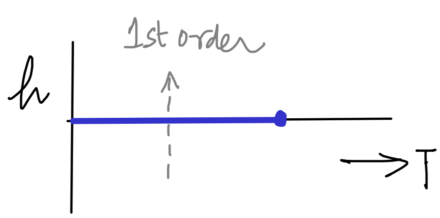
374°C, 218 atm, 0.323 g/cm³

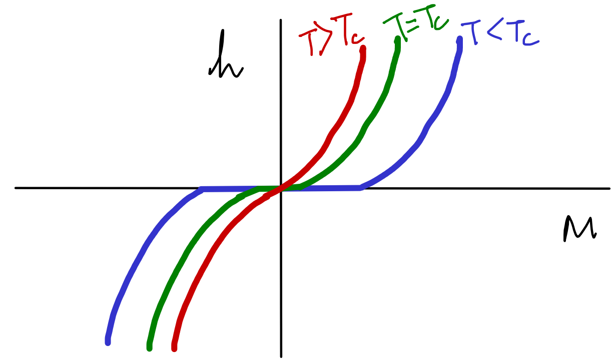
② Ferromagnet below T_c:

There is a first order phase transition as the magnetic field crosses zero.

The magnetization jumps from a finite value to another finite value with an opposite sign as the magnetic field changes sign.

This jump in magnetization vanishes at the critical temperature, T_c.



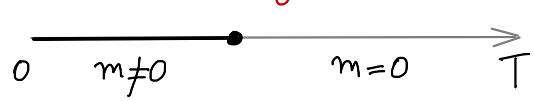


Continuous Phase Transition

This is a phase transition in which there is no discontinuity in the order parameter. The order parameter is zero at the transition point (also known as a **critical point**) and is also zero on one side of the transition.

Examples:

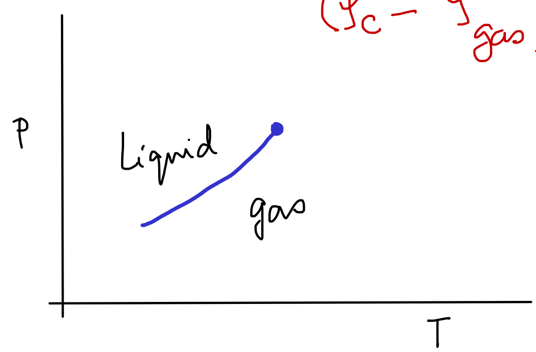
① Transition from a ferromagnet to a paramagnet at $h=0$.
Order parameter — Magnetization



② Transition at the liquid-gas critical point.

Order parameter: $(\rho_{liq} - \rho_c)$ on the liquid side of the coexistence line

$(\rho_c - \rho_{gas})$ on the gas side of the coexistence line



<u>Universality</u>: The vanishing of the order parameter and the divergences of various response functions at a critical point are associated with exponents, whose values are identical for critical points in very different systems. These exponents are known as **critical exponents**.

Systems with identical sets of exponents fall into the same **universality class**. Such systems can be described by the same theory in the vicinity of the critical point.

The critical exponents $\alpha, \beta, \gamma, \delta, \eta, \nu$

<u>Notation</u>: $f(x) \sim g(x)$ as $x \to x_0$

$$\Rightarrow \lim_{x \to x_0} \frac{f(x)}{g(x)} = \text{constant}$$

E.g. $\sin kx \sim x$ as $x \to 0$

$e^{ax} - 1 \sim x$ as $x \to 0$

$$\frac{x + a}{2x^2 - b} \sim \frac{1}{x} \quad \text{as } x \to \infty$$

Reduced temperature, $t \equiv \dfrac{T - T_c}{T_c}$

$T \to T_c \iff t \to 0$

Vanishing of the order parameter with varying temperature:

$$M \sim |t|^{\beta}, \quad h=0$$

Divergence of the heat capacity

$$C \sim |t|^{-\alpha}$$

Divergence of the susceptibility

$$\chi \sim |t|^{-\gamma}$$

Vanishing of the order parameter with varying field:

$$M \sim |h|^{1/\delta}, \quad T=T_c$$

Correlation function, $G(\vec{r}) = \langle m(\vec{r}_0+\vec{r})\, m(\vec{r}_0)\rangle - \langle m(\vec{r}_0)\rangle^2$

For $h=0$ and $t \to 0$, $G(\vec{r}) \sim \dfrac{e^{-r/\xi}}{r^{d-2+\eta}}$,

where d is the number of spatial dimensions and η is the anomalous critical exponent. ξ is the correlation length

Divergence of the correlation length:

$$\xi \sim |t|^{-\nu}$$

<u>Scaling Relations</u>:

Even though the values of the critical exponents are different for systems falling in different universality classes, certain relations between them are valid across universality classes.

These are known as scaling relations.

$\gamma = \nu(2-\eta)$ — Fisher

$\alpha + 2\beta + \gamma = 2$ — Rushbrooke

$\gamma = \beta(\delta - 1)$ — Widom

$\nu d = 2 - \alpha$ — Josephson