PHSDSC717P – Mathematical Methods IV Lab

Saurish Chakrabarty

Acharya Prafulla Chandra College

Part I
Assignments

1 Complex Roots

(a) Using the two-dimensional Newton-Raphson method, find the cube roots of a complex number by forming two simultaneous equations from the real and imaginary parts.

(b) Calculate the roots by de Moivre's formula and compare the results.

(c) Find complex roots of an arbitrary algebraic equation.

Hints and Relevant Python Functions:

- For obtaining the modulus and argument of a complex number, you may use `cmath.polar`.

Part II
Theory

1 Two-dimensional Newton-Raphson Method

Ref: Press et. al., *Numerical Recipes in Fortran*
Problem: Find a root of $\vec{F}(\vec{x}) = 0$ using a guess value $\vec{x_0}$.
Expanding $\vec{F}(\vec{x})$ in a Taylor series to linear order, we have,

$$\vec{F}(\vec{x} + \delta\vec{x}) = \vec{F}(\vec{x}) + J(\vec{x})\,\delta\vec{x}, \tag{1}$$

where $J(\vec{x})$ is the Jacobian matrix, i.e., $J_{ij} = \dfrac{\partial F_i}{\partial x_j}$. If $\vec{x} + \delta\vec{x}$ is a root, then the left hand side of the above equation must vanish. Then, we have,

$$J\delta\vec{x} = -\vec{F} \Rightarrow \delta\vec{x} = -J^{-1}\vec{F}. \tag{2}$$

Thus, the algorithm for finding the root is to change $\vec{x}$ following this rule until convergence is achieved.

$$\delta\vec{x}_{n+1} = \vec{x}_n - J^{-1}\vec{F}. \tag{3}$$

For a two-dimensional problem, J is a 2×2 matrix. Thus, if,

$$J(\vec{x}) = \begin{pmatrix} a(\vec{x}) & b(\vec{x}) \\ c(\vec{x}) & d(\vec{x}) \end{pmatrix}, \text{ then, } J^{-1}(\vec{x}) = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}. \tag{4}$$

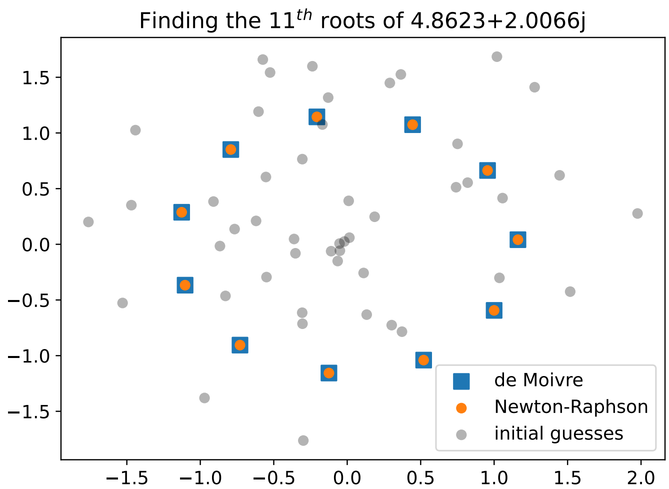

Figure 1: Sample calculation of roots with two-dimesional Newton-Raphson method and comparison with exact answers.